S.6 APPLIED MATHEMATICS

TEST 3, TERM 2 2019

NUMERICAL METHODS

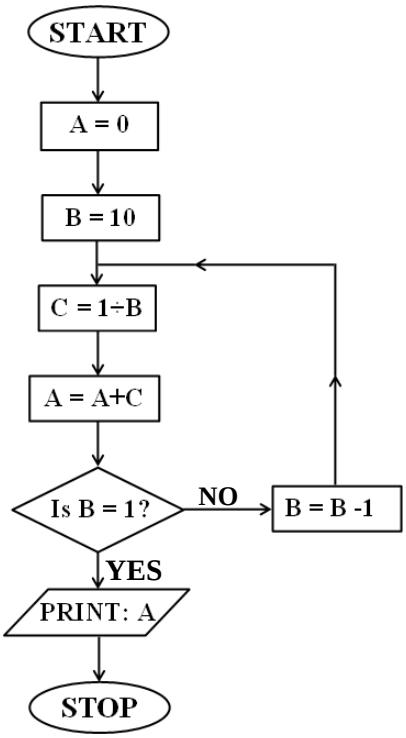
Time: 1 hour 30 minutes

Attempt ***all*** questions.

1. Given below are the values of $f(x)$ for corresponding value of x.
   $f(0.4) = -0.9613$, $f(0.6) = -0.5103$ and $f(0.8) = -0.2231$
   Use linear interpolation to determine;
   i) $f(0.7)$ correct to 4 decimal places, (*03 marks*)
   ii) $f^{-1}(-0.4308)$ correct to 2 decimal places. (*02 marks*)

2. Given that $y = sin\theta$ and θ is measured with a maximum possible error of 2%. If $\theta = 30^o$ determine the;
   (i) absolute error in y, (*02 marks*)
   (ii) interval with in which the value of y lies. Give your answer correct to 4 significant figures. (*03 marks*)

3. Study the flowchart below and answer the questions that follow.



   (i) Perform a dry run for the above flow chart.
   (ii) Suggest a purpose of the flow chart.

(*05 marks*)

4. a) Use the trapezium rule with six ordinates to find the approximate value of $\int_2^5 xe^{-x}dx$ correct to ***three*** significant figures.

   b) Find the area bounded by the curve $y = xe^{-x}$ between $x = 2$ and $x = 5$.

   c) Find the percentage error in (a) above.

   (*12 marks*)

5. The numbers **A** and **B** are rounded off to ***a*** and ***b*** with errors $\boldsymbol{e_1}$ and $\boldsymbol{e_2}$ respectively.

   a) Show that the absolute relative error in the product **AB** is given by;

   $\frac{|a||e_2|+|b||e_1|}{ab}$ (*05 marks*)

   b) Given that A = 6.43 and B = 37.2 are rounded off to the given number of decimal places indicated;

   i) State the maximum possible errors in A and B. (*02 marks*)

   ii) Determine the absolute error in AB. (*02 marks*)

   iii) Find the limits with in which the product AB lies. Give your answer to 4 decimal places. (*03 marks*)

6. Given the equation; $x^3 - 6x^2 + 9x + 2 = 0$.

   a) Find graphically the root of the equation which lies between -1 and 0.

   (*05 marks*)

   b) i) Show that the Newton Raphson formula for approximating the root of the equation is given by $x_{n+1} = \frac{2}{3}\left\{\frac{{x_n}^3 - 3{x_n}^2 - 1}{{x_n}^2 - 4x_n + 3}\right\}$ where $n = 0, 1, 2, \ldots$

   (*03 marks*)

   ii) Use the formula in b(i) above, with an initial approximation in a) above to find the root of the given equation correct to two decimal places.

   (*04 marks*)

END